Energy conditions and current acceleration of the universe

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The energy conditions provide a very promising model-independent study of the current acceleration of the universe. However, in order to connect these conditions with observations, one often needs first to integrate them, and then find the corresponding constraints on some observational variables, such as the distance modulus. Those integral forms can be misleading, and great caution is needed when one interprets them physically. A typical example is that the transition point of the deceleration parameter $q(z)$ is at about $z \simeq 0.76$ in the ΛCDM model. However, with the same model when we consider the dimensionless Hubble parameter $E(z)$, which involves the integration of $q(z)$, we find that $E(z)$ does not cross the line of $q(z) = 0$ before $z = 2$. Therefore, to get the correct result, we cannot use the latter to determine the transition point. With these in mind, we carefully study the constraints from the energy conditions, and find that, among other things, the current observational data indeed strongly indicate that our universe has ocne experienced an accelerating expansion phase between the epoch of galaxy formation and the present.

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I. INTRODUCTION

Ever since the discovery of the accelerated expansion of the universe by the supernova (SN) Ia observations [1], many efforts have been made to understand the mechanism of this accelerated expansion. Although different observations all pointed to the existence of dark energy [2, 3, 4, 5, 6], the nature of it is still a mystery. For recent review of dark energy models, one may refer to [7].

Due to the lack of satisfactory dark energy models, many model-independent methods were proposed to study the properties of dark energy and the geometry of the universe [8, 9, 10, 11]. In particular, in the reconstruction of the deceleration parameter $q(z)$, it was found that the strongest evidence of acceleration happens at the redshift $z \sim 0.2$ [8, 9]. The sweet spot of the equation of state parameter $w(z)$ was found to be around the redshift $z \sim 0.2 - 0.5$ [9, 10].

Another very interesting and model-independent approach is to consider the energy conditions [12]. Since these conditions do not require a specific equation of state of the matter in the universe, they provide very simple and model-independent bounds on the behavior of the (total) energy density, pressure and look-back time as a function of red shift. As a matter of fact, even before the discovery of the acceleration of the universe, studies of these conditions already led Visser in 1997 to conclude correctly that current observations suggest that the "strong energy condition" (SEC) was violated sometime between the epoch of galaxy formation and the present. This implies that no possible combination of "normal" matter is capable of fitting the observational data [13]. Santos *et al* [14] further investigated these conditions and found that all the energy conditions seem to have been violated in a recent past of the cosmic evolution. On the other hand, assuming that the universe is flat and contains only dark matter and dark energy, Sen and Scherrer studied the constraints of the weak energy condition (WEC) on the evolution of the Hubble parameter and the coordinate distance, and obtained an upper bound on Ω_m [15]. As the authors themselves pointed out, this bound is generic and independent of the nature of the dark energy. Lately, we also investigated these conditions, and applied them to the 192 essence supernova Ia data [16]. In particular, we showed that the universe had once experienced an accelerated expansion period. From the degeneracy of the distance modulus at low redshift, we also argued that the choice of w_0 for probing the property of dark energy is misleading. One explicit example was used to support this argument.

In this paper we would like to point out that, while such an approach is very promising, one has to use these energy conditions with great caution. This is mainly because these conditions are local in terms of the expansion factor $a(t)$, and when we use them to study their constraints on some observational variables, such as the distance modulus $\mu(z)$, we often need to consider their integral forms. Such integrated formulas can be misleading, and result in wrong interpretations. To see this clearly, let us consider a function $f(x)$, which is smooth enough so that the integral $I(x) = \int_0^x f(x')dx'$ exists. Obviously, if $f(x) \geq 0$ for $x \in (0, x_s)$, we must have $I(x) \geq 0$ for $x \in (0, x_s)$ [Fig. 1(a)]. However, the inverse does not hold, that is the condition $I(x) \geq 0$ for $x \in (0, x_s)$ does not imply $f(x) \geq 0$ for $x \in (0, x_s)$. In particular, it does

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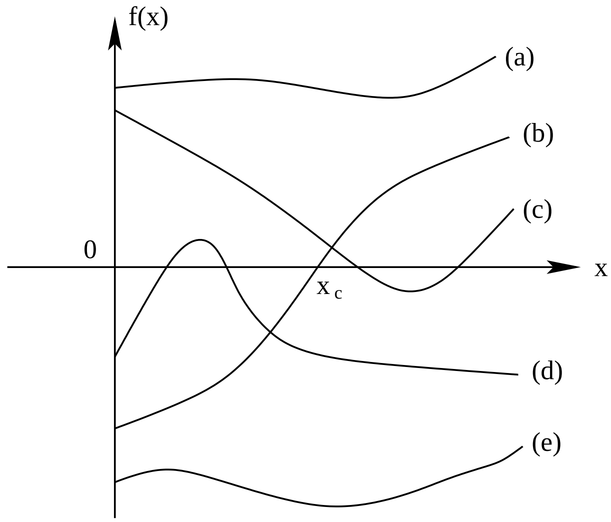

FIG. 1: The function $f(x)$ for several different cases. In Cases (a) and (c), the integral $I(x) = \int_0^x f(x')dx'$ is always non-negative, while in Cases (d) and (e) it is always non-positive. In Case (b), $f(x)$ has a crossing point at $x = x_c$, but the crossing point of $I(x)$ is much great than x_c.

not exclude the possibility that $f(x)$ can be negative for some values of $x \in (0, x_s)$. Case (c) in Fig. 1 shows explicitly this possibility. In fact, all what we can conclude from $I(x) \geq 0$ is that $f(x)$ must be non-negative for certain value(s) of $x \in (0, x_s)$. Similarly, if $f(x) \leq 0$ for $x \in (0, x_s)$, we must have $I(x) \leq 0$ for $x \in (0, x_s)$ [Fig. 1(e)], but the inverse is in general not true [Fig. 1(d)].

Another important remark is that the crossing points of $f(x)$ and $I(x)$ can be quite different. For example, $f(x)$ has a crossing point at x_c along the curve (b) in Fig. 1, but clearly the crossing point of $I(x)$ must be much greater than x_c.

With all of the above in mind, let us consider the constraints that the energy conditions impose. In particular, the paper is organized as follows. In section II, we consider all the four energy conditions, and apply them first to the ΛCDM model and then to a fiducial model. For the ΛCDM model, Fig. 2 shows clearly that the deceleration parameter $q(z)$ passes the transition point at $z \simeq 0.76$. However, with the same model when we consider the dimensionless Hubble parameter $E(z)$, which involves the integration of $q(z)$, we find that $E(z)$ does not cross the line of $q(z) = 0$ before $z = 2$. Of course, the latter does not mean that the transition must have happened at $z > 2$. Similar results can be obtained from our fiducial model given by Eq.(12), where $q(z)$ is negative during the period $0.1 < z < 0.15$. But, Fig. 3 shows that $E(z)$ never crosses the line of $q(z) = 0$. Applying our arguments to the 192 essence SN Ia data, in section III we find that the data indeed strongly indicate that our universe has once experienced an accelerating expansion phase between the epoch of galaxy formation and the present. In section IV we conclude the paper with some discussions.

II. ENERGY CONDITIONS

The energy conditions can be expressed as [13, 14]

$$\text{NEC} \Leftrightarrow \rho + p \geq 0, \tag{1}$$
$$\text{WEC} \Leftrightarrow \rho \geq 0 \text{ and } \rho + p \geq 0, \tag{2}$$
$$\text{SEC} \Leftrightarrow \rho + 3p \geq 0 \text{ and } \rho + p \geq 0, \tag{3}$$
$$\text{DEC} \Leftrightarrow \rho \geq 0 \text{ and } \rho \pm p \geq 0. \tag{4}$$

Combining with the FRW equation, for an expanding universe the SEC requires that

$$\rho + 3p \geq 0 \Leftrightarrow q(t) = -\ddot{a}/(aH^2) \geq 0, \tag{5}$$
$$\rho + p \geq 0 \Leftrightarrow \dot{H} - \frac{k}{a^2} \leq 0. \tag{6}$$

The Hubble parameter $H(t) = \dot{a}/a$ and the deceleration parameter $q(t)$ are related by,

$$H(z) = H_0 \exp\left[\int_0^z [1 + q(u)] d\ln(1+u)\right], \tag{7}$$

where the subscript 0 means the current value of the variable. Substituting Eq. (5) into Eq. (7), we find

$$H(z) \geq H_0(1+z). \tag{8}$$

On the other hand, the integration of Eq. (6) yields

$$H(z) \geq H_0\sqrt{1 - \Omega_k + \Omega_k(1+z)^2}, \tag{9}$$

for redshift $z = a_0/a - 1 \geq 0$. For $z \geq 0$, Eq. (8) implies Eq. (9). So we conclude that

$$\text{SEC} \Rightarrow H(z) \geq H_0(1+z), \tag{10}$$
$$\text{NEC} \Rightarrow H(z) \geq H_0\sqrt{1 - \Omega_k + \Omega_k(1+z)^2}. \tag{11}$$

However, the converses of Eqs. (10) and (11) are not true. In particular, if Eq. (8) is satisfied, it does not mean that the SEC had never been violated, because Eq. (8) is the integration of Eq. (5), similar to Case (c) illustrated in Fig. 1. In this case what we know is that the SEC had once been satisfied. But, if the bound (8) is violated, then it is sure that the SEC had once been violated. By virtue of the same reasoning, the satisfaction of Eq. (9) does not mean that the NEC had *never* been violated, but does mean that the NEC had *once* been satisfied. Likewise, if this bound is violated, then the NEC had *once* been violated.

These conclusions are very important, and we would like to use two specific examples to help us further understand these key results. The first example is the flat ΛCDM model with $\Omega_m = 0.27$. The flat ΛCDM model has accelerated expansion up to redshift $z = 0.76$ and decelerated expansion for $z > 0.76$. We plot the evolution of the deceleration parameter in Fig. 2 where it clearly shows that $q(z)$ passes its transition point at

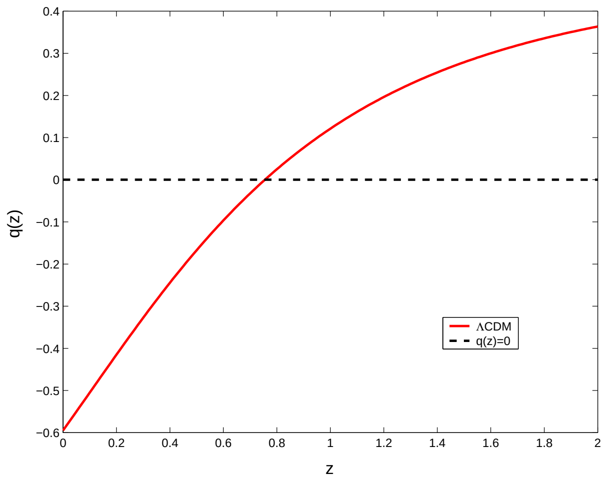

FIG. 2: The evolution of the deceleration parameter. The solid line is for the flat ΛCDM model with $\Omega_m = 0.27$ and the dashed line is for $q(z) = 0$.

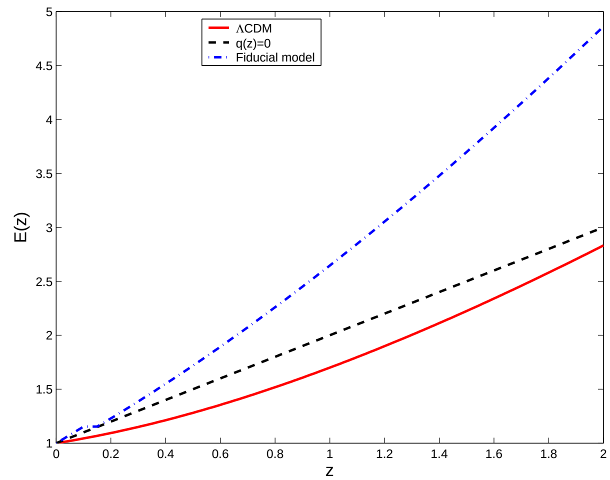

FIG. 3: The evolution of the dimensionless Hubble parameter $E(z)$. The solid line is for the flat ΛCDM model with $\Omega_m = 0.27$, the dashed line is for $q(z) = 0$, and the dash dotted line is for the Fiducial model (12).

$z \simeq 0.76$. We also plot the evolution of the dimensionless Hubble parameter $E(z) = H(z)/H_0$ for the same model in Fig. 3. Even though when $z \geq 0.76$, $q(z) \geq 0$, we still have $H(z) \leq H_0(1+z)$ up to $z \sim 2$ [16]. This may seem very strange, but it can be easily understood through Fig. 4, where we plot the difference of the function $(1+q)/(1+z)$ between the ΛCDM model and the zero-acceleration model $q(z) = 0$. Because the Hubble parameter is related with the deceleration parameter $q(z)$ by Eq. (7), $H(z)$ is an integral of $q(z)$. Therefore, the shaded area gives the value of $\ln(H_2/H_1)$, where H_2 denotes the Hubble parameter of the ΛCDM model and H_1 denotes the Hubble parameter of the model with $q(z) = 0$. The positive area of $2 \geq z \geq 0.76$ does not compensate the negative area of $z < 0.76$, so the total area is negative up to $z \sim 2$. This explains why $E(z) \leq 1 + z$ for the ΛCDM model even up to $z = 2$.

The second example is the fiducial model

$$q(z) = \begin{cases} 1/2, & z \leq 0.1, \\ -1, & 0.1 < z < 0.15, \\ 1/2, & z \geq 0.15. \end{cases} \quad (12)$$

Substituting this model into Eq. (7), we obtain

$$E(z) = \begin{cases} (1+z)^{3/2}, & z \leq 0.1, \\ 1.1^{3/2}, & 0.1 \leq z \leq 0.15, \\ [(1.1/1.15)(1+z)]^{3/2}, & z \geq 0.15. \end{cases} \quad (13)$$

The evolution of $E(z)$ for the fiducial model is shown in Fig. 3 by the dash dotted line. We see that even the bound (8) is satisfied for any given $z \geq 0$, $q(z)$ can still be negative in the interval $0.1 \leq z \leq 0.15$. Thus, the bound (8) does not exclude the possibility that the universe had once experienced an accelerating expansion phase. From this condition what we can really conclude is that the universe had once experienced a decelerating expansion phase.

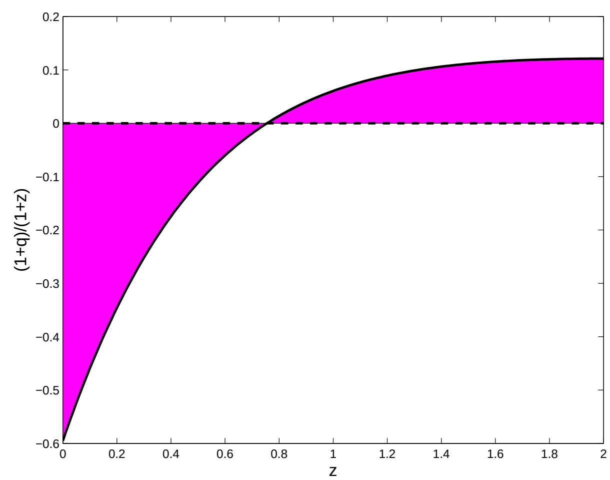

FIG. 4: The difference of the function $(1+q(z))/(1+z)$ between the flat ΛCDM model and the model with $q(z) = 0$.

The ΛCDM model and the fiducial model (12) clearly show that we must be very careful with the interpretation of the bounds (8) and (9) derived from the energy conditions. If the bound (8) is satisfied, then we conclude that the SEC was once satisfied, although it is not necessarily always satisfied. The fiducial model (12) shows clearly that even if the bound (8) is satisfied, the SEC can still be violated during a certain period of time. If the bound (8) is violated, what we are sure is that the SEC was once violated (but not necessarily always violated). The ΛCDM model shows that even if the bound (8) is violated, the SEC can still be satisfied for $z > 0.76$. Likewise, if the bound (9) is satisfied, then we are confident that NEC was once satisfied (but not necessarily

always satisfied). If the bound (9) is violated, then we are confident that NEC was once violated.

III. COSMOLOGICAL APPLICATIONS OF THE ENERGY CONDITIONS

Now, let us consider the bounds on the luminosity distance. This was already discussed in [9]. Here we would like to emphasize the key points derived in the last section. We consider only the flat universe. Then, the luminosity distance is given by

$$d_L(z) = (1+z)\int_0^z \frac{dz'}{H(z')}. \tag{14}$$

The extinction-corrected distance modulus is $\mu(z) = 5\log_{10}[d_L(z)/\mathrm{Mpc}] + 25$. Substituting Eqs. (8) and (9) into Eq. (14), we obtain the upper bounds on the luminosity distance

$$H_0 d_L(z) \le z(1+z), \tag{15}$$

$$H_0 d_L(z) \le (1+z)\ln(1+z). \tag{16}$$

To compare these bounds with the 192 essence SN Ia data [4], we plot them in the distance modulus redshift graph in Fig. 5. The region under the lower solid line corresponds to the bound (15) and the region under the upper solid line corresponds to the bound (16). If all or some of the SN Ia data are inside the region under the lower solid line, it means that the universe had once experienced a decelerated expansion phase. If some or all of the SN Ia data are outside the region under the lower solid line, it means the universe had once accelerated. From Fig. 5, we see that some SN Ia are indeed outside the region under the lower solid line, so it is evident that the universe had once experienced an accelerated expansion. Note that due to the integration effect, even if some high z SN Ia data are outside the region under the lower solid line, it does not mean that we have evidence of an accelerating expansion in the high z region, as shown in Figs. 2 and 3. Even if almost all the SN Ia data are outside the region under the lower solid line, it does not mean there is no evidence for past deceleration.

If all or some of the SN Ia data are inside the region under the upper solid line, it means that the universe had once not experienced a super-accelerated expansion. If some or all of the SN Ia data are outside the region under the upper solid line, it means the universe has once experienced a super-accelerated expansion. Since the SN Ia data are in the region bounded by the two solid lines, we conclude that *the universe had once experienced an accelerating expansion phase, and the acceleration is not always super-acceleration*. But this does not mean that the universe has never experienced a period of super-accelerated or decelerated expansion.

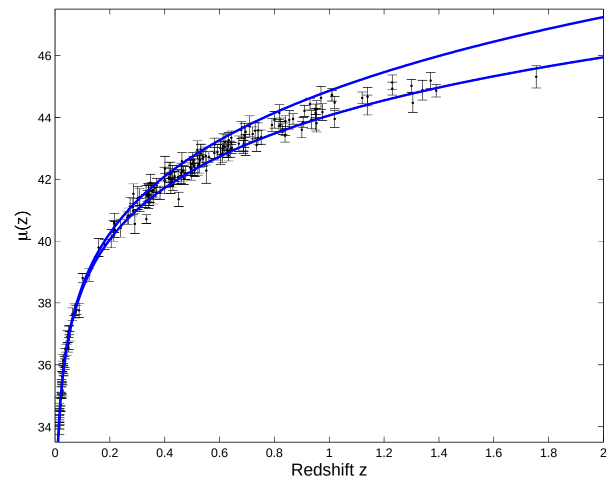

FIG. 5: The distance modulus $\mu(z)$. The solid lines correspond to the bounds from the SEC condition and NEC condition.

Now, let us turn to the bounds on the age of the universe derived from the energy conditions. The age of the universe is

$$t_0 = \int_0^\infty \frac{dz}{(1+z)H(z)}. \tag{17}$$

Substituting Eq. (8) into Eq. (17), we get

$$H_0 t_0 \le 1. \tag{18}$$

From the NEC condition for a flat universe, we get $H_0 t_0 < \infty$. The current observational values for t_0 and H_0 are $t_0 = 13.7^{+0.1}_{-0.2}$ and $H_0 = 0.73^{+0.04}_{-0.03} \times (9.78\mathrm{Gyr})^{-1}$. Because $H_0^{-1} = 13.4^{+0.6}_{-0.7}$, so the current age of the universe is consistent with the bound (18). However, this does not mean that the current age of the universe is compatible with the SEC. The only conclusion we can derive from this bound is that the SEC once held during the past of the evolution of the universe.

If dark energy component satisfies the SEC, then we find

$$E^2(z) = H^2(z)/H_0^2 \ge \Omega_m(1+z)^3 + (1-\Omega_m)(1+z)^2, \tag{19}$$

which results in

$$\Omega_m \le \frac{E^2(z) - (1+z)^2}{z(1+z)^2}. \tag{20}$$

The results of $H(z)$ from [17], $H(1.53) = 140 \pm 14$, yield the upper bound $\Omega_m \le -0.28 \pm 0.08$. This upper bound is clearly violated by current observations. Therefore, we conclude that SEC must have once been violated. In other words, the universe had once experienced an accelerated expansion.

It is interesting to note that the WEC requires [15]

$$\Omega_m \le \left.\frac{E^2(z) - 1}{(1+z)^3 - 1}\right|_{z=1.53} = 0.18 \pm 0.05, \tag{21}$$

which is also a little bit lower than that given by recent observations [1, 2, 3, 4, 5, 6].

IV. DISCUSSION

The energy conditions $\rho + 3p \geq 0$ and $\rho + p \geq 0$ give lower bounds (8) and (9) on the Hubble parameter $H(z)$, and upper bounds on the distance modulus $\mu(z)$. If some SN Ia data are outside the region bounded by Eq. (8), then we conclude that the universe had once experienced an accelerated expansion. If some SN Ia data are outside the region bounded by Eq. (9), then we can tell that the universe had once experienced a super-accelerated expansion. In other words, the distance modulus-redshift graph can be used to provide direct model-independent evidence of accelerated and super-accelerated expansion. Therefore, the energy conditions provide direct and model-independent evidence of the once-accelerated expansion phase. The bounds on the distance modulus also provide some directions for the future SN Ia observations. In particular, they can give some bounds on the age of the universe and bounds on the distance modulus-red shift graph.

Unfortunately, the method has also its own limitations. For example, it does not provide us with any detailed information about the acceleration, nor the nature of dark energy. In addition, Because the luminosity distance is an integral of the Hubble parameter, the distance modulus does not give us useful information about the exact transition point of the universe from decelerated expansion to accelerated expansion.

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